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Decomposition Symmetric Group S_5 to Character Table Subgroup by Using Young Subgroups of Type $S_3 \times S_2$ and $S_2 \times S_2 \times S_1$

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Abstract: In this paper we will work with the decomposition character table for symmetric groupsS_4 to character table of all subgroups by using a new young subgroup of type$S_2 \times S_2$.

Keywords: Symmetric group, Young subgroup, wheel, primitive wheel, (classical &Relative) character.

I. INTRODUCTION

In 2009 may (Richard Bayley) Introduced a new idea on the subject ,concept and who knows relative character table , and in 2010 a research has been submitted by Aladdin and Haider.B.A with title of division of symmetric group S_5 partial factions ,where the mentioned research involve the using of same concept. Also in 2011 the same researchers whose names meutioned above presented relative character table for S_8 by using young subgroup of type $S_7 \times S_1$, Bassim .K.M. and Haider .B.A. presnted a research with title of "costractioncharcter table of a symmetric group$S_4$by using permutation module " in 2013. In the present work I found out a new approach of “decomposition character table for symmetric groups S_4 for all subgroups by using a new young subgroup of type$S_2 \times S_2$”, see [2],[3]and[4].

II. BASIC THEORY

Definition (2.1): [5]
A young Diagram is a finite collection of boxes arranged in left – justified rows with the row site monotonically decreasing .The young diagram associated to the partition $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_r)$ is the one that has r rows and $\lambda_i$boxes on the ith row.

Definition (2.2) :[3]
A standard (young) tableau a young tableaux whose entries are increasing a erases each row and each column.

For example let$\lambda = (3,2,1)$then standard young tableaux are:

III. YOUNG TABLEAUX PROCEDURE:

Definition (3.1): [1]
A partition of a positive integer n is a sequence of positive integers $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$ satisfying $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_k > 0$ and $n = \lambda_1 + \lambda_2 + \cdots + \lambda_k$ we write $\lambda \vdash n$ to denoted that λ is a partition of n .

Definition (3.2): [9]
A Youngdiagram is a finite collection of boxes arranged in left-justified rows with the row size monotonicly descreasing, the Young diagram associated to the partition $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$ is the one that has k rows and λ_i boxes on the ith row.

Definition (3.3): [9]
Suppose $\lambda \vdash n$,A Young tableau t of shape λ, is obtained by filling in the boxes of a Young diagram of λ with 1,2,..,n in away that each number occurring exactly once. In this case, we say that t is a λ-tableau.

Definition (3.4): [8]
A semi standard Young tableau is a way of filling the boxes in a Young diagram with positive integers so that the entries are weakly increase in rows, strictly increase down columns.

Definition (3.5): [1]
A standard Young tableau is a Young tableaux whose entries are strictly increasing across each row and column.

IV. H-CONJUGACY CLASSES

Definition (4.1): [7]
In a group G, two elements g and h are called conjugate when
$g = g.h.g^{-1}$ for some g in G.

Definition (4.2) :[7]
Conjugation can be extended from elements to subgroups. If $H \subset G$ is a subgroup and$g \in G$, the setg.

$$H.g^{-1} = \{g.h.g^{-1} : h \in H\},$$

is a subgroup of G, called H-conjugate subgroup to H.

Definition (4.3): [6,7]
Let $\bigcup_{i=1}^{k} \lambda_i = \{\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k\}$ be a partition of into k disjoint subsets. Then the corresponding Young subgroup of S_n, the symmetric group on n letters, is the subgroup

$$S_{\lambda 1} \times S_{\lambda 2} \times \dots \times S_{\lambda k}$$

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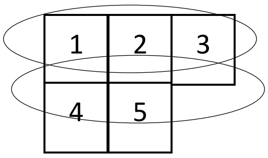
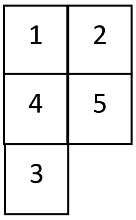
Definition (4.5):
Let$S_{\lambda 1} \times S_{\lambda 2} \times \ldots \times S_{\lambda k}$ be young-subgroup, a partition of into k disjoint subsets. We say that every$S_{\lambda i}$ is orbital i, and denoted for any element belong in $S_{\lambda i}$ by [i].

Labelling H-conjugacy classes (4.6):
Since the young subgroup is $H_y = S_3 \times S_2$:



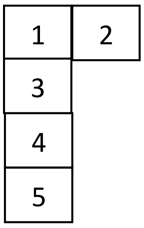
$$\begin{cases} \{1,2,3\} \to \text{Orbital number } [1] \\ \{4,5\} \to \text{Orbital number } [2] \end{cases}$$

For $H_y = S_2 \times S_2 \times S_1$:



$$\begin{cases} \{1,2\} \to \mathit{Orbital\ number}\ [1] \\ \{3,4\} \to \mathit{Orbital\ number}\ [2] \\ \{5\} \to \mathit{Orbital\ number}\ [3] \end{cases}$$

For $H_y = S_2 \times S_1 \times S_1 \times S_1$:

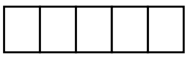


$$\begin{cases} \{1,2\} \to \mathit{Orbital\ number}\ [1] \\ \{3\} \to \mathit{Orbital\ number}\ [2] \\ \{4\} \to \mathit{Orbital\ number}\ [3] \\ \{5\} \to \mathit{Orbital\ number}\ [4] \end{cases}$$

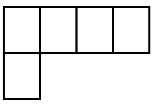
Definition (4.7):
Let c be the permutation we can define [c] by Appling the labeling map f to each of the entries of c we call that result is a wheel.

Definition (4.8):
A wheel [c] is said to be primitive wheel whenever all repeated sequences in the permutation c and repeated wheels are removed from the permutation but one if existed, we denoted it [p.w].for examples:

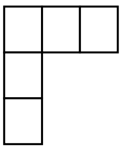
Calculation H-conjugacy classes forS_5 :
Let $H_y = S_3 \times S_2$ be the young subgroup of S_5 then they H-conjugacy classes are: conjugate class wheel primitive wheel P.V.F



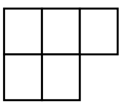
$\{[11122]\ \{[11122]\ \{[11122] \to (1)$



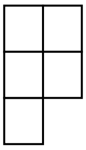
$$\begin{cases} [1112][2] \\ [1122][1] \end{cases} \begin{cases} [1112][2] \\ [1122][1] \end{cases} \begin{cases} [1112] \to (1), [2] \to (1) \\ [1122] \to (1), [1] \to (1) \end{cases}$$



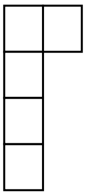
$$\begin{cases} [111][2][2] \\ [112][1][2] \\ [122][1][1] \end{cases} \begin{cases} [1][2][2] \\ [112][1][2] \\ [122][1][1] \end{cases} \begin{cases} [1] \to (123), [2] \to (1)(2) \\ [112] \to (1), [1] \to (1), [2] \to (1) \\ [122] \to (1), [1] \to (1)(2) \end{cases}$$



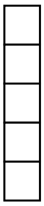
$$\begin{cases} [111][22] \\ [112][12] \\ [122][11] \end{cases} \begin{cases} [1][2] \\ [112][12] \\ [122][1] \end{cases} \begin{cases} [1] \to (123), [2] \to (12) \\ [112] \to (1), [12] \to (1) \\ [122] \to (1), [1] \to (12) \end{cases}$$



$$\begin{cases}[11][12][2]\\ [12][12][1]\end{cases}\begin{cases}[1][12][2]\\ [12][12][1]\end{cases}\begin{cases}[1]\to(12),[12]\to(1),[2]\to(1)\\ [12]\to(1)(2),[1]\to(1)\end{cases}$$

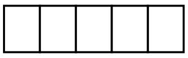


$$\begin{cases}[11][1][2][2]\\ [12][1][1][2]\end{cases}\begin{cases}[1][1][2][2]\\ [12][1][1][2]\end{cases}\begin{cases}[1]\to(12)(3),[2]\to(1)(2)\\ [12]\to(1),[1]\to(1)(2),[2]\to(1)\end{cases}$$

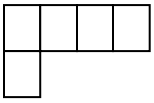


$$\{[1][1][1][2][2]\{[1][1][1][2][2]\{[1]\to(1)(2)(3),[2]\to(1)(2)$$

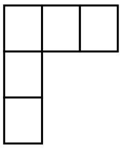
Let $H_y = S_2 \times S_2 \times S_1$ be the young subgroup of S_5 then they H-conjugacy classes are: conjugate class wheel primitive wheel P.V.F



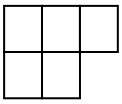
$$\{[11223]\ \ \{[11223]\quad [11223]$$



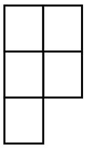
$$\begin{cases}[1122][3]\\ [1123][2]\\ [1223][1]\end{cases}\begin{cases}[1122][3]\\ [1123][2]\\ [1223][1]\end{cases}\begin{cases}[1122]\to(1)\quad [3]\to(1)\\ [1123]\to(1)\quad [2]\to(1)\\ [1223]\to(1)\quad [1]\to(1)\end{cases}$$



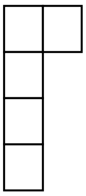
$$\begin{cases}[112][2][3]\\ [113][2][2]\\ [123][1][2]\end{cases}\begin{cases}[112][2][3]\\ [113][2][2]\\ [123][1][2]\end{cases}\begin{cases}[112]\to(1)\quad [2]\to(1)\quad [3]\to(1)\\ [113]\to(1)[2]\to(1)(2)\\ [123]\to(1)\quad [1]\to(1)\quad [2]\to(1)\end{cases}$$



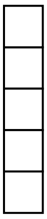
$$\begin{cases}[112][23]\\ [113][22]\\ [122][13]\\ [123][13]\end{cases}\begin{cases}[112][23]\\ [112][12]\\ [122][13]\\ [123][13]\end{cases}\begin{cases}[112]\to(1)\ [23]\to(1)\\ [112]\to(1)\ [12]\to(1)\\ [122]\to(1),[13]\to(1)\\ [123]\to(1),[13]\to(1)\end{cases}$$



$$\begin{cases}[11][22][3]\\ [12][12][3]\\ [11][23][2]\\ [12][13][2]\end{cases}\begin{cases}[1][2][3]\\ [12][12][3]\\ [1][23][2]\\ [12][13][2]\end{cases}\begin{cases}[1]\to(12),[2]\to(12),[3]\to(1)\\ [12]\to(1)(2),\ [3]\to(1)\\ [1]\to(12),[23]\to(1),\ [2]\to(1)\\ [12]\to(1),\ [13]\to(1),\ [2]\to(1)\end{cases}$$



$$\begin{cases}[11][2][2][3]\\ [12][1][2][3]\\ [13][1][2][2]\end{cases}\begin{cases}[1][2][2][3]\\ [12][1][2][3]\\ [13][1][2][2]\end{cases}\begin{cases}[1]\to(12),[2]\to(1)(2),[3]\to(1)\\ [12]\to(1)\ [1]\to(1)\ [2]\to(1)\ [3]\to(1)\\ [13]\to(1)\ [1]\to(1)\ [2]\to(1)(2)\end{cases}$$



$$\{[1][1][2][2][3]\{[1][1][2][2][3]\{[1]\to(1)(2)\,,[2]\to(1)(2),[3]\to(1)$$

Example (4.9):
Let (123) (45) permutation belong in S_5, and $H_y = S_3 \times S_2$ from by using labeling H-conjugacy class , we get on wheel [w]=[111][22], the sequence in w is 111 and 22 so the associated primitive wheel is [p.w]= [1][2].

Partition Valued Function(PVF) formula:
Let [p.w] be a primitive wheel and is a H-conjugate class :

$$HCL: [P.w] \rightarrow \pi$$

whereπ is a permutation then the $f_{(PVF)}$ partitions is called permutation Valued Function (PVF), defined by the following :

$$P.V.F = \frac{length\ [w]}{length\ [p.w]}$$

For example (4.9) let [w]=[111][22] of two cycles in S_5& [p.w] =[1][2] then the partition valued function is :

$$P.V.F = \frac{length\ [w]}{length\ [p.w]} = \frac{[111]}{[1]} = \frac{3}{1} = 3$$
$$P.V.F = \frac{length\ [w]}{length\ [p.w]} = \frac{[22]}{[2]} = \frac{2}{1} = 2$$

Then
HCL: [1] $\rightarrow \pi$since P.V.F =3, then [1]→(123) permutation belong S_3
HCL: [2] $\rightarrow \pi$since P.V.F =2, then [1]→(12) permutation belong S_2
HCL= (123),(12)

V. MAIN RESULT

In this section is devoted to study the character table S_4 and to find the character table subgroup by using young subgroup of type $S_2 \times S_2$

Theorem:
If S_4 is symmetric group of order 24 and the young subgroup is $S_2 \times S_2$ then the relative character table for S_4 is decomposition to all subgroups

1- If the young subgroup of typ$S_3 \times S_2$ then the relative character table for S_5be:

$$\equiv (RS_5) = (\equiv S_3) \oplus (\equiv S_2) \oplus \sum_{i=1}^{7} \oplus_i (\equiv S_1).$$

2- If the young sub group of type $S_3 \times S_2$ then the relative character table for S_5 be:

$$\equiv (RS_5) = (\equiv S_2 \oplus \equiv S_2) \oplus \sum_{i=1}^{14} \oplus_i (\equiv S_1)$$

VI. CONCLUSIONS

In the end of the paper we managed to analysis of symmetric S_4 to the subgroups by using a new young subgroup and this was evident from character table, which explains this work can assume crisis represents an atom or molecule and that the process of fission, is nothing but the process of segmentation the group to subgroup.

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